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Multi-response Permutation Procedure as An Alternative to
the Analysis of Variance: An SPSS Implementation

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Abstract

A permutation test can free the researchers from stringent parametric assumptions. Multi-Response Permutation Procedure (MRPP) is a class of multivariate permutation tests of group differences that offers vast improvement over the traditional analysis of variance based tests in terms of robustness under non-normal data and heterogeneous error variances. However, psychologists seldom make use of MRPP in data analysis. It is partly attributable to the fact that MRPP is not implemented in popular statistical packages that psychologists use. A set of SPSS macros implementing the MRPP test is provided in this paper that may prove useful for psychological researchers analyzing experimental data.

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Permutation tests represent the ideal situations where one can derive the exact probabilities associated with a test statistic, rather than approximate values obtained from common probability distributions, such as the t , F and χ^2 , when some or all the parametric assumptions are relaxed (Mielke & Berry, 2001, p. 1). Carrying out randomization of the collected data rather than relying on the parametric assumptions of random sampling or normality also enhances the practicability of a test. In many cases, the only assumption made by a permutation test is the exchangeability of the observational units under the null hypothesis, which is much weaker than the parametric assumptions (see e.g., Hayes, 1996, for an excellent discussion of this issue). Extended discussions of the merits of permutation tests can be found in book-length treatments elsewhere (e.g. Edgington, 1995; Good, 1993).

Multi-response Permutation Procedure (MRPP) was first introduced by Mielke, Berry, and Johnson (1976) as a technique for detecting the difference between *a priori* classified groups. It turned out to be an extremely versatile data-analytic framework from which a number of applications fall out, such as the measurement of agreement, multivariate correlation and association coefficients, and the detection of autocorrelation (see Mielke & Berry, 2001 for a complete coverage of applications of the MRPP framework).

MRPP is often analogous to parametric tests such as the t test or the analysis of variance. In fact, it has been shown that many “classical” tests are special cases of MRPP. For instance, Mielke and Berry (1994) demonstrated the equivalence

between members of the MRPP family of statistics and the *ANOVA* / *MANOVA* test statistics. What makes MRPP more attractive than the parametric counterparts is its robustness under violations of the parametric assumptions (see e.g., Mielke & Berry, 1994).

Despite its advantages over traditional methods, MRPP is not widely used in psychology. I suspect that part of the reason is the lack of implementation of this method in commercial statistical software packages. Although FORTRAN programs were available from the inventors of this method (e.g. Mielke & Berry, 2001), none of the popular statistical packages, such as SPSS and SAS, offers an option to perform the MRPP tests ¹. Perhaps this explains why MRPP is relatively unknown to many psychologists. A citation index search shows that practicing psychological researchers are not citing the seminal papers on the MRPP technique, an indication that they are not using this method. In this paper I amend the situation by providing a set of SPSS macros that can be invoked just as easily as any other SPSS routines such as the familiar GLM or ONEWAY procedures when analyzing multivariate data arising from experimental studies.

The remainder of this paper is organized in the following manner. First, I shall briefly introduce the MRPP framework and the many analyses opportunities using MRPP. Next, I shall describe the SPSS macros in some detail. Finally, I shall analyze real data sets using the macro to demonstrate its user-friendliness.

Overview of MRPP

I shall start by presenting the basic formulations of MRPP for a simple one-way layout. Only the minimum amount of details are included. A fuller

description of MRPP can be found in Mielke (1984).

A Basic Formulation

It is assumed that there are k groups (or treatments) in a fixed-effects one-way design ². The outcome measures are possibly multivariate. The total sample size is N , and each group is of size n_i , where $i = 1, \dots, k$, such that $\sum_{i=1}^k n_i = N$. Let $\mathbf{y}_I = (y_{1I}, \dots, y_{rI})'$ denote an $r \times 1$ vector of (possibly non-commensurate) multivariate response observations from subject I , where $I = 1, \dots, N$.

If the multivariate responses are not commensurate, i.e., not measured in the same unit, they must be standardized to the same metric before any MRPP analyses can proceed. If r is one, the experimental design is simply a univariate one-way layout, then commensuration is not needed, but if r is greater than one, commensuration becomes a must. I will elaborate on the issue of commensuration later. For the moment, it is assumed that the outcome measures are indeed commensurate.

Let S_i denote a set that contains all the \mathbf{y}_I 's belonging to the i th group, where $i = 1, \dots, k$. The basic building block of an MRPP statistic is a distance function between two commensurate multivariate response vectors, \mathbf{y}_I and \mathbf{y}_J ,

$$\Delta_{I,J} = \left[\sum_{h=1}^r (y_{hI} - y_{hJ})^2 \right]^{v/2}. \quad (1)$$

Equation (1) is a special case of the Minkowski family of distance functions often used in multidimensional scaling (see e.g., MacCallum, 1988). The two most commonly used distance functions are: 1) the non-metric squared Euclidean distance

$$\Delta_{I,J} = \sum_{h=1}^r (y_{hI} - y_{hJ})^2, \quad (2)$$

obtained by setting $v = 2$ in Equation (1), and 2) the metric Euclidean distance

$$\Delta_{I,J} = \left[\sum_{h=1}^r (y_{hI} - y_{hJ})^2 \right]^{1/2}, \quad (3)$$

obtained by setting $v = 1$ in Equation (1). Clearly, both Euclidean and squared Euclidean distance functions can be written compactly in matrix notation as

$$\Delta_{I,J} = \left[(\mathbf{y}_I - \mathbf{y}_J)'(\mathbf{y}_I - \mathbf{y}_J) \right]^{v/2}. \quad (4)$$

When $v = 1$, MRPP tests based on the metric Euclidean distance has the nice property of being more robust and less influenced by outliers (Mielke & Berry, 1994). The use of the non-metric squared Euclidean distance leads to a less robust test. However, $v = 2$ often yields easier interpretations of the test results, because many popular tests, such as the t test, the *ANOVA F*, and Hotelling's T^2 all involve the use of squared distance.

The MRPP statistic is a weighted average of within-group distances, where the weights are determined by the group sizes³. Geometrically, a smaller value of the MRPP statistic indicates higher within group concentration and larger between group differences (Mielke, 1984). Therefore, in terms of detecting treatment effect or between group differences, a smaller value of the MRPP statistic is necessarily "better."

The MRPP statistic is given by

$$\delta_{obs} = \sum_{i=1}^k C_i \xi_i, \quad (5)$$

where C_i is the group weight for $i = 1, \dots, k$, such that $\sum_{i=1}^k C_i = 1$, and

$$\xi_i = \binom{n_i}{2}^{-1} \sum_{I < J} \Delta_{I,J} \Psi_{S_i}(\mathbf{y}_I) \Psi_{S_i}(\mathbf{y}_J), \quad (6)$$

is the average within-group distance for the i th group. The summation is taken over all integer values of I and J such that $1 \leq I < J \leq N$, and $\Psi_{S_i}(\mathbf{y}_I)$ is an indicator function given by

$$\Psi_{S_i}(\mathbf{y}_I) = \begin{cases} 1, & \text{if } \mathbf{y}_I \in S_i, \\ 0, & \text{otherwise.} \end{cases}$$

Mielke (1984) gives an extensive discussion on the choice of group weights. For Euclidean distance, $C_i = n_i/N$ should be used in general, and for squared Euclidean distance, $C_i = (n_i - 1)/(N - k)$ should be used.

The test of significance of δ_{obs} is carried out by assuming the null hypothesis that the

$$M = \frac{N!}{\prod_{i=1}^k n_i!},$$

possible ways of allocating the N responses into the k groups are equally probable.

In other words, the null hypothesis states that the outcome observations are independent of group membership. Since each permutation yields a realized value of δ , the significance level of δ_{obs} is determined by the number of times the realized δ 's are smaller than or equal to δ_{obs} . Hence, the p -value associated with δ_{obs} is $\Pr(\delta_{obs}) = \{\#\ \delta \leq \delta_{obs}\}/M$.

Because M is usually a very large number even for relatively small sample sizes, the exact null distribution of the MRPP statistic is difficult to obtain. Therefore, Mielke et al. (1976) proposed the method of using a Pearson Type III curve to approximate the discrete MRPP null distribution. This method only needs the first three moments of the MRPP null distribution. The exact mean, variance,

and skewness of the null distribution of the MRPP statistic are defined as

$$\begin{aligned}\mu_\delta &= M^{-1} \sum_{i=1}^M \delta_i, \\ \sigma_\delta^2 &= M^{-1} \sum_{i=1}^M \delta_i^2 - \mu_\delta^2, \\ \gamma_\delta &= \frac{M^{-1} \sum_{i=1}^M \delta_i^3 - 3\mu_\delta \sigma_\delta^2 - \mu_\delta^3}{\sigma_\delta^3}.\end{aligned}$$

Efficient computational formulae for finding these moments can be found in Mielke et al. (1976). A standardized statistic T can then be calculated as

$$T = \frac{\delta_{obs} - \mu_\delta}{\sigma_\delta}, \quad (7)$$

and because T is a one-to-one function of δ_{obs} , the p -value associated with δ_{obs} is approximated easily by numerically integrating the Pearson Type III density function from negative infinity to T

$$\Pr(\delta_{obs}) = \Pr(T) \doteq \int_{-\infty}^T f(x|\gamma_\delta) dx, \quad (8)$$

where $f(x)$ is the Pearson Type III density function, completely characterized by a skewness parameter (see Johnson, Kotz, & Balakrishnan, 1994, for details of the Type III curve).

Percentage points of the Pearson Type III distribution have been extensively tabulated (Harter, 1967), but these tables are rarely known to psychologists. Fortunately, it turns out that there is a direct correspondence between the χ^2 distribution and the Pearson Type III distribution. Because most statistical packages, including SPSS, has a built-in capability to calculate the cumulative distribution function (CDF) of the χ^2 distribution, the approximate p -value for the

MRPP statistic becomes easy to obtain. According to Harter (1967), when the skewness parameter is positive, the Pearson Type III density can be recognized as a standardized χ^2 density with degrees of freedom $\nu = 8/\gamma^2$, mean $\mu = \nu$, and variance $\sigma^2 = 2\nu$, where γ is the skewness of the Pearson Type III curve. Therefore, whenever $\gamma > 0$, the p -value associated with T is approximated as $\Pr(T) = \Pr(X \leq x)$, where X is a χ^2 random variable with ν degrees of freedom, and $x = \mu + T\sigma$. In MRPP applications, the skewness is often negative. Therefore, the p -value in Equation (8) can be approximated using the compliment of the χ^2 CDF, $\Pr(T) = 1 - \Pr(X \leq x)$, where X is a χ^2 random variable with ν degrees of freedom, and $x = \mu - T\sigma$. When the skewness is zero, the Pearson Type III distribution becomes a standard normal distribution, and the p -value of T can be easily obtained from the standard normal CDF.

Commensuration

When the number of dimensions r in the outcome observations is greater than one, the dependent variables may be expressed in completely different units, so a multivariate standardization problem arises. Mielke and Berry (1994) proposed two methods of dealing with the commensuration problem. Both methods involve a slight change of the distance function defined in Equation (4).

The first method is termed Euclidean commensuration, and is intended for $v = 1$. Returning to the notations used in Equation (1), let $\mathbf{y}_I = (y_{1I}, \dots, y_{rI})'$ denote an $r \times 1$ vector ($r \geq 2$) of non-commensurate multivariate response observations from subject I . The corresponding vector of commensurate multivariate observations $\mathbf{x}_I = (x_{1I}, \dots, x_{rI})'$ can be found by pre-multiplying the vector \mathbf{y}_I by an $r \times r$ matrix $\mathbf{\Phi}^{-1/2}$, i.e., $\mathbf{x}_I = \mathbf{\Phi}^{-1/2}\mathbf{y}_I$, where $\mathbf{\Phi} = \text{diag} [\phi_j^2]$ denotes

a diagonal matrix with elements ϕ_j^2 on the diagonal for $j = 1, \dots, r$. The ϕ_j 's are defined as

$$\phi_j = \sum_{I < J} |y_{jI} - y_{jJ}|.$$

Combining the above results with Equation (4), the Euclidean commensuration essentially changes the definition of the Euclidean distance function to

$$\Delta_{I,J} = \left[(\mathbf{y}_I - \mathbf{y}_J)' \mathbf{\Phi}^{-1} (\mathbf{y}_I - \mathbf{y}_J) \right]^{1/2}. \quad (9)$$

The second method is called Hotelling commensuration, to be used with the squared Euclidean distance ($v = 2$). It changes the definition of the distance function to

$$\Delta_{I,J} = (\mathbf{y}_I - \mathbf{y}_J)' \mathbf{S}^{-1} (\mathbf{y}_I - \mathbf{y}_J), \quad (10)$$

where

$$\mathbf{S} = \frac{1}{N} \begin{bmatrix} \sum_{I=1}^N (y_{1I} - \bar{y}_{1.})^2 & \cdots & \sum_{I=1}^N (y_{1I} - \bar{y}_{1.})(y_{rI} - \bar{y}_{r.}) \\ \vdots & & \vdots \\ \sum_{I=1}^N (y_{rI} - \bar{y}_{r.})(y_{1I} - \bar{y}_{1.}) & \cdots & \sum_{I=1}^N (y_{rI} - \bar{y}_{r.})^2 \end{bmatrix}$$

is the $r \times r$ sample variance-covariance matrix.

Relation to Existing Statistics

When $r = 1$, $v = 2$, and group weights are $C_i = (n_i - 1)/(N - k)$, the MRPP statistic δ_{obs} has the following relation to the mean square treatment used in ANOVA (Mielke & Berry, 1994):

$$\delta_{obs} = \frac{(N - 1)\mu_{\delta} - 2(k - 1)MS_{trt}}{N - k},$$

where

$$MS_{trt} = \frac{1}{k-1} \sum_{i=1}^k n_i (\bar{y}_i - \bar{y}),$$

and \bar{y}_i is the mean of the i th group, and \bar{y} is the grand mean for the combined sample. Because μ_δ is fixed for a given dataset, and MS_{trt} depends only on the means, the statistical inference based on MRPP is not influenced by variance heterogeneity (Mielke & Berry, 1994).

When $r \geq 2$, $v = 2$, group weights are $C_i = (n_i - 1)/(N - k)$, and Hotelling commensuration is used, the MRPP statistic is related to the Bartlett-Nanda-Pillai trace criterion V in *MANOVA* by the following identity

$$\delta_{obs} = \frac{2(r - V)}{N - k}. \quad (11)$$

In addition, V is also directly related to Hotelling's T^2 statistic and Mahalanobis D^2 statistic when the number of groups is 2. It is well known that T^2 is a generalization of the Student's t test. Therefore these parametric tests are all special cases of the MRPP test, yet MRPP does not make use of normal distribution theory to make inferences. Considering the fact that psychological data are rarely normal (Micceri, 1989), MRPP-based analyses offers a vast improvement over traditional methods.

MRPP Regression Analyses of Linear Models

The MRPP test, as described in the previous section, is only applicable to one-way designs. To analyze data arising from more complex experimental designs, the MRPP is combined with regression models with appropriately reparameterized design matrices. This procedure is called MRPP regression analyses of linear models (Mielke & Berry, 2001).

It is well known that the F test of an effect in factorial analysis of variance models can be approached within the ordinary least squares (OLS) regression framework by comparing a reduced model (without the effect being analyzed) to a full model (with the effect). For example, a fixed-effects one-way layout posits a full model of the form

$$y_{ij} = \mu + \alpha_i + e_{ij}, \quad (12)$$

where y_{ij} denotes the response observation from the j th subject ($j = 1, \dots, n_i$) in the i th group ($i = 1, \dots, k$), μ is the grand mean, α_i is the i th treatment effect, and e_{ij} is the error term. To analyze the treatment effect, a reduced model is given by

$$y_{ij} = \mu + e_{ij}. \quad (13)$$

In the regression analysis, the treatment effect can be coded into the design matrix as $k - 1$ columns, and the grand mean is simply represented as a column of ones. To obtain the *ANOVA* F statistic for the effect α , one would first fit the model in Equation (13), and then add the $k - 1$ variables representing the α effect to the regression equation to fit the full model in Equation (12). The F test of incremental R^2 from the reduced model to the full model is equivalent to the F statistic derived using *ANOVA*.

The MRPP regression analyses uses the same rationale, except that it replaces the parametric test of incremental fit by the MRPP test. For the preceding one-way layout, one would first fit the reduced model, obtain the regression coefficients, and compute the residuals using the regression equation. Then, an MRPP analysis of the residuals provides a statistical test of the treatment effect. More generally, MRPP regression analyses is a three-step procedure. In step one, one would fit a reduced regression model with all the effects in the regression equation but the one

being analyzed. In step two, residuals are computed from the reduced model. This is followed by step three, an MRPP analysis of the residuals. For example, consider a two-way factorial design, and the experimenter is interested in analyzing the interaction effect between the two factors. The full model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk}, \quad (14)$$

where the α 's and β 's are the main effects, and $(\alpha\beta)_{ij}$ represents the interaction.

The reduced model without the interaction is given by

$$y_{ijk} = \mu + \alpha_i + \beta_j + e_{ijk}. \quad (15)$$

The MRPP analysis of residuals from fitting the reduced model in Equation (15) is a test of the interaction effect. Note that the MRPP test requires grouping information, but for all experimental designs, grouping information is implicit. For example, for a 2-by-2 factorial design, there are altogether four treatment combinations. All that is needed in the MRPP analysis of residuals is a single grouping variable indicating to which treatment combination a particular residual belongs.

Mielke and Berry (2001) suggested that instead of using residuals from OLS regression, one can more profitably use residuals from Least Absolute Deviation (LAD) regression. Unlike OLS regression that estimates the regression coefficients by minimizing squared deviations, LAD regression minimizes absolute values of deviations. Mielke and Berry (2001) showed that MRPP analyses of LAD residuals using the Euclidean distance function ($v = 1$) leads to a more robust test against outliers.

Description of The Macros

Two SPSS macros are provided. The macros are programmed using SPSS's matrix language. The first macro is a fully functional MRPP routine that uses the Pearson Type III approximation to obtain p -values. The second macro is a simple LAD regression program that uses an algorithm tailored after the downhill simplex method (Press, Teukolsky, Vetterling, & Flannery, 1992) to obtain LAD regression coefficients. The downhill simplex takes starting values from an OLS regression. Together, the two macros can perform all the tests described in the preceding sections. The macros can be downloaded in electronic form from the following website: <http://www.unc.edu/~lcai/programs.html>.

As is true for all SPSS macro programs, one only need to run the them once in each SPSS session. As soon as the macros are run, they become part of the SPSS system, and can be invoked just as any other SPSS procedures. The macros will stay active until the user quits SPSS. The macros read analysis data set from the working data file. Only limited error checking is provided, so if the macro prints error messages, discard the output and do not interpret the results. The missing data is deleted list-wise.

Macro MRPP

The syntax of the MRPP macro is as follows

```
MRPP dist=[1|2] comm=[0|1|2] weight=[1|2] iv=grp dvs=dep vars.
```

The option “dist” determines the type of distance function to be used in the MRPP analysis, where 1 indicates Euclidean distance ($v = 1$), and 2 for squared Euclidean distance ($v = 2$). The default value is 1.

The option “comm” determines the type of multivariate commensuration to be applied on the outcome observations, where 0 means no commensuration, 1 means Euclidean commensuration, and 2 means Hotelling commensuration. The default value is 1. However, this option has no effect when the outcome observations are univariate, because commensuration is not necessary for the case of $r = 1$.

The option “weight” determines the type of group weights used in computing the MRPP test statistic. When it is set to 1, weights of the form $C_i = n_i/N$ will be used, and when it is set to 2, weights of the form $C_i = (n_i - 1)/(N - k)$ will be used. The default value is 1.

The user must specify the name of the grouping (independent) variable in the required argument “iv.” The dependent variables should be listed in the last argument “dvs.” The names of the dependent variables must be separated by blank spaces.

For example, suppose the SPSS working data file contains data from a one-way design, and the dependent variables are called “y1,” “y2,” and “y3,” and the grouping variable is called “grp.” The following syntax

```
MRPP iv=grp dvs=y1 y2 y3.
```

conducts an MRPP analysis using Euclidean distance, Euclidean commensuration, and the first type of group weights. Note that by omitting the options, their values defaults to 1's. The following syntax

```
MRPP dist=2 comm=2 weight=2 iv=grp dvs=y1 y2 y3.
```

conducts an MRPP analysis using squared Euclidean distance, Hotelling commensuration, and the second type of group weights. This combination of options essentially gives a permutation version of the multivariate analysis of variance.

Macro LADREG

The syntax of the LADREG macro is

```
LADREG ftol=[1E-6] maxit=[200] const=[1] dv=dv ivs=predictors.
```

The option “ftol” sets the convergence criterion for the iterative algorithm. The default value is 1E-6, in scientific notation.

The option “maxit” sets the maximum number of iterations allowed in the iterative line fitting algorithm before it quits without reaching the minimum. The default is 200.

The option “const” determines whether the intercept is estimated or not. When it is set to 1, a column of ones is included in the design matrix. When it is set to 0, the intercept is not estimated. The default value is 1.

The required argument “dv” takes the name of the dependent variable. The dependent variable must be in the SPSS working data file.

The names of the predictors are entered through the argument “predictors.” The names must be separated by blank spaces.

For instance, if the dependent variable is called “y” in the open data set, and the the predictors are called “x1” and “x2,” the following code

```
LADREG dv=y ivs=x1 x2.
```

conducts an LAD regression analysis with the intercept.

Examples

In this section, I will analyze two example data sets using the SPSS macros provided in the last section. The output will be provided and the relation of the

MRPP analyses to their parametric counterparts will be explored. The reader can download the SPSS syntax file generating the output from the same web address as the macro download page.

MRPP Analyses of Multivariate Outcomes

The data set comes from Mielke et al. (1976). In the original manuscript, it is listed in Table I on page 1419. The data set contains three outcome variables, called “v1,” “v2,” and “v3.” The grouping variable is called “grp.” There are $k = 4$ groups, and the total N is 21. The design is not balanced, and the group sizes (n_i) are 5, 4, 8, and 4 for $i = 1, \dots, k$, in that order. The command

```
mrpp dist=1 comm=0 weight=1 iv=grp dvs=v1 v2 v3.
```

conducts an MRPP analysis using Euclidean distance and the first type of group weight. No commensuration is requested, as the original analysis in Mielke et al. (1976) does not use any kind of commensuration. The above command replicates Mielke et al.’s (1976) analysis.

The output from the macro is in Figure 1. The first few lines print some information about the options. It is seen that no commensuration is invoked, and the Euclidean distance is used. The Group Summary table prints the group size (n_i), average within group distance (ξ_i), and the group weights (C_i). The MRPP statistic $\delta_{obs} = \sum_{i=1}^k \xi_i C_i = 3.45$ is located underneath “Tests of Significance” as “Delta.” Then the exact moments of the MRPP null distribution are printed as Mean ($\mu_\delta = 8.11$), Variance ($\sigma^2 = .31$), and Skewness ($\gamma = -1.06$). The standardized statistic T is found to be $(\delta_{obs} - \mu_\delta)/\sigma = (3.45 - 8.11)/\sqrt{.311} = -8.35$. which gives a p -value of 2.52E-6. Assuming that the three outcome measures are commensurate,

there is indication of large between group difference. These results match exactly with the output obtained from the BLOSSOM package (Cade & Richards, 2001).

A slightly different command

```
mrpp dist=1 comm=1 weight=1 iv=grp dvs=v1 v2 v3.
```

conducts the same analysis but with Euclidean commensuration turned on. The output is in Figure 2. With a standardized statistic of -8.18 and a p -value of 3.34E-6, the macro produced exactly the same results as BLOSSOM (Cade & Richards, 2001).

The following command

```
mrpp dist=2 comm=2 weight=2 iv=grp dvs=v1 v2 v3.
```

uses the squared Euclidean distance with Hotelling commensuration and the second type of group weights for the same one-way design. The output is in Figure 3. Again, the results match exactly with the BLOSSOM package (Cade & Richards, 2001). As noted afore, this command essentially produces a permutation version of the multivariate analysis of variance. In this case, the Bartlett-Nanda-Pillai trace criterion V for the test of group difference is equal to 1.161. The direct correspondence between the MRPP test statistic and V is evident because according to Equation (11), δ_{obs} should be equal to

$$\frac{2(r - V)}{N - k} = \frac{2(3 - 1.161)}{21 - 4} = .216,$$

which is indeed the case. For this data set, the parametric test V translates into an approximate F statistic, $F(9, 51) = 3.577$, $p = .002$. Judging from the rather small p -values obtained from both parametric test and permutation test, it seems that the groups do differ significantly.

MRPP Regression Analysis of LAD Residuals

In this section I will analyze a data set from the popular experimental design textbook by Keppel (1991). This data set characterizes a 2×3 balanced factorial design. Factor A has two levels, and factor B has three levels. The cell size is 4, so the total N is 24, and the number of treatment combinations is 6. In this example, I am going to analyze the interaction effect using both the LADREG macro and the MRPP macro.

First, the full and reduced models are already given by Equations (14) and (15). For a 2×3 design, 3 columns of effects code are needed to fully represent the two main effects. In the data set, the outcome variable is called “y,” the effects coded variable for factor A is called “x1,” and the two coded variables to represent factor B are called “x2” and “x3.” In addition, I also created a six-level grouping variable (called “tc”) to indicate to which treatment combination the outcome observation belongs. In the next step, “x1,” “x2,” and “x3” are entered into the LAD regression. This is the reduced model, and LAD residuals are computed from the reduced model coefficients. In the final step, I run the MRPP macro with the residuals as the outcome variable, and “tc” as the grouping variable.

The following command

```
LADREG dv=y ivs=x1 x2 x3.
```

fits the reduced model to the data. The output is in Figure 4. It is seen that the OLS solution is also the LAD solution, because the first row of the Starting Simplex is the same the first row in the Final Simplex. It took the macro 5 iterations to determine that it has reached a minima, and at this minimum, the sum of absolute

values of the residuals is 94⁴. To compute the residuals using the LAD regression coefficient estimates, the following SPSS commands can be used

```
compute res = y-(10.00000-x1-3*x2+x3).  
execute.
```

For the MRPP analysis of residuals, the following command is used

```
mrpp iv=tc dvs=res.
```

and the output is in Figure 5. The MRPP p -value for the $A \times B$ interaction is .46. In contrast, the conventional ANOVA analysis of this interaction effect gives $F(2, 18) = 3.92$, $p = .038$, which is significant at the .05 level.

Discussion

The MRPP a robust alternative to the traditional normal theory based parametric tests, such as the t test and the analysis of variance. However, MRPP is not widely known to psychological researchers, and part of the reason is that it has not been incorporated into major statistical packages. To overcome this limitation, SPSS macros that implement the MRPP test are provided in this paper, in the hope that they will make MRPP more accessible to psychologists analyzing experimental data. It is shown by analyzing real data sets that the macros are as easy to use as the standard SPSS procedures such as GLM or ONEWAY.

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Footnotes

¹A free stand-alone software (BLOSSOM; Cade & Richards, 2001) is available from <http://www.fort.usgs.gov/products/software/blossom/blossom.asp>. However, users still face data management problems in transporting their data sets into formats amenable to analysis in BLOSSOM.

²The notations are primarily formatted after Mielke and Berry (2001) to achieve consistency.

³For notational simplicity both the “excess group” and the truncation of distance to a preset maximum are excluded from discussion in the present paper (for details see Mielke & Berry, 2001).

⁴Note that the LAD coefficients are not unique.

Figure Captions

Figure 1. MRPP Analysis – I

Figure 2. MRPP Analysis – II

Figure 3. MRPP Analysis – III

Figure 4. LADREG Output

Figure 5. MRPP Analysis of LAD Residuals

Multi-Response Permutation Procedure (MRPP)

No commensuration invoked

Metric Euclidean distance function invoked

Total sample size

21

Group summary

	N	Distance	Weights
	5.000000000000	3.580166225429	.238095238095
	4.000000000000	3.214880033153	.190476190476
	8.000000000000	3.681188834369	.380952380952
	4.000000000000	3.060205742503	.190476190476

Moments of the MRPP null distribution

	Mean	Variance	Skewness
	8.110161833941	.311462970802	-1.05618948865

Test of Significance

	Delta	T	Sig.
	3.450032614510	-8.35015906239	.000002517390

Multi-Response Permutation Procedure (MRPP)

Euclidean commensuration invoked: print the Phi matrix

820836	0	0
0	698896	0
0	0	1102500

Metric Euclidean distance function invoked

Total sample size

21

Group summary

	N	Distance	Weights
	5.000000000000	.003893040759	.238095238095
	4.000000000000	.003491150118	.190476190476
	8.000000000000	.004050703860	.380952380952
	4.000000000000	.003412210188	.190476190476

Moments of the MRPP null distribution

	Mean	Variance	Skewness
	.008679002569	.000000357445	-1.05825645899

Test of Significance

	Delta	T	Sig.
	.003784965519	-8.18583042093	.000003343974

Multi-Response Permutation Procedure (MRPP)

Hotelling Commensuration invoked: print SSCP matrix

270.5714286	225.5714286	277.8571429
225.5714286	233.2380952	264.1904762
277.8571429	264.1904762	370.9523810

Squared Euclidean distance function invoked

Total sample size

21

Group summary

	N	Distance	Weights
	5.000000000000	.130895548347	.235294117647
	4.000000000000	.242633791183	.176470588235
	8.000000000000	.289113143558	.411764705882
	4.000000000000	.134329829458	.176470588235

Moments of the MRPP null distribution

	Mean	Variance	Skewness
	.300000000000	.000441890947	-.515294791321

Test of Significance

	Delta	T	Sig.
	.216368532954	-3.97843405036	.000729406618

Least Absolute Deviation (LAD) Regression Using
Downhill Simplex Minimization Method

Criterion variable is:

Y

Sample Size

24

Starting Simplex:

10.00000	-1.00000	-3.00000	1.000000
11.00000	-1.00000	-3.00000	1.000000
10.00000	.000000	-3.00000	1.000000
10.00000	-1.00000	-2.00000	1.000000
10.00000	-1.00000	-3.00000	2.000000

Final Simplex:

10.00000	-1.00000	-3.00000	1.000000
11.00000	-1.00000	-3.00000	1.000000
10.63281	-1.24219	-2.60156	.578125
10.34912	-1.00244	-2.67139	.975586
10.41016	-1.52734	-3.01953	1.726563

Number of iterations:

5.00

Sum of absolute values of the residuals:

94.00

LAD Parameter Estimates

Intercep	X1	X2	X3
10.00000	-1.00000	-3.00000	1.000000

Multi-Response Permutation Procedure (MRPP)

Commensuration not needed: dimensionality of response = 1

Metric Euclidean distance function invoked

Total sample size

24

Group summary

	N	Distance	Weights
	4.000000000000	4.000000000000	.166666666667
	4.000000000000	5.000000000000	.166666666667
	4.000000000000	6.000000000000	.166666666667
	4.000000000000	7.000000000000	.166666666667
	4.000000000000	5.000000000000	.166666666667
	4.000000000000	5.000000000000	.166666666667

Moments of the MRPP null distribution

	Mean	Variance	Skewness
	5.318840579710	.275221989148	-.754307128559

Test of Significance

	Delta	T	Sig.
	5.333333333333	.027625448948	.460765917774